

Problem Solving Through History

Problem Set #9

1. Find the probability that in a room with 5 people at least two will have the same birthday.
2. In the game of Keno described in class, find the probability that you have exactly six matches.
3. Suppose you and a friend are playing the game of tossing a coin where you win if 10 heads occur before 10 tails do (and your friend wins if 10 tails occur before 10 heads do). Because of Thanksgiving break, you stop the game when you have 8 heads and your friend has 5 tails. How should you divide up the prize of \$1000?
4. Some has a weighted coin that lands heads-up with probability $\frac{2}{3}$ and tails up with probability $\frac{1}{3}$. If heads comes up you pay \$1; if tails comes up you receive \$1.50. What is the expected value of this game?
5. You roll a fair die. If 1 comes up you win \$25. If 2 comes up you win \$5. If 4 or 5 come up, you lose \$10. If 6 comes up you lose \$15. What is the expected value of this game?
6. Pepsi runs a contest whereby under the cap of one in a million bottles there is a message saying you won \$1,000,000. Suppose the two-liter bottle costs \$2. What is the expected value of buying a bottle (be sure to include the cost of the bottle).
7. A lottery offers a grand prize of \$100,000, three second prizes of \$10,000, and 10 third prizes of \$1,000. The winning ticket can be any 6-digit number from 000,000 to 999,999. If a person buys one ticket, what is a fair price to pay for the ticket?
8. Toss two dice and sum the up-faces.
 - a) What are the odds in favor of obtaining 7?
 - b) Against obtaining 6?
9. Draw one card from 52-card deck. What are the odds of drawing:
 - a) a King or a diamond?
 - b) Against drawing an ace?
10. Find the probability that an event E will occur if the odds are 8 to 3 in favor of E.

11. Find the probability that an event E will occur if the odds are 3 to 5 against E.

12. A player will win \$18 if she throws a double on the first toss of a pair of dice.

a) What are the odds of her winning?

b) What is a fair price to pay to play this game?

13. If the distance traveled by an object after t seconds is $t^2 - 3t$ feet, what is the instantaneous velocity of the object when $t = 4$? Use the limit definition to find the function rule for the derivative, do not use derivative rules.

14. If the distance traveled by an object after t seconds is $3t + 6$, what is the instantaneous velocity when Use the limit definition to find the function rule for the derivative, do not use derivative rules.)

a) $t = 2$?

b) $t = 5$?

15. Find the following limits (if they exist)

a) $\lim_{x \rightarrow -1} \frac{x^2 - 3x - 4}{x + 1}$

b) $\lim_{x \rightarrow 0} \frac{x^2}{x}$

c) $\lim_{x \rightarrow 0} \frac{x}{x^2}$

d) $\lim_{x \rightarrow \infty} \frac{2x^2 - 3x + 100}{7000 - 225x - 5x^2}$

e) $\lim_{x \rightarrow \infty} \frac{1700x - 30}{4 + 3x^2}$

16. Find the slope of the line tangent to the curve $f(x) = 4 - 3x^2$ at the point $(1,1)$.

17. Find the slope of the line tangent to the curve $f(x) = x^2 - 2x + 4$ at the point $(-1,7)$.