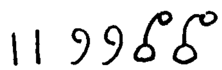
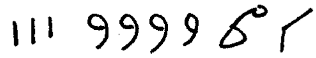
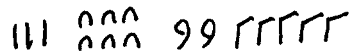
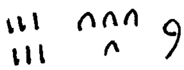
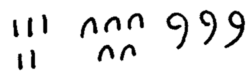
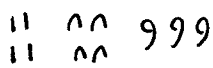
Problem Solving Through History
Problem Set #1

1. Perform the following operations and express your answer in both hieroglyphics and our system.

a) Add  and 

b) Add  and 

c) Subtract  from 

d) Subtract  from 

2. Use Duplation to find a) 28×35 b) 106×72 c) 101×101

3. Use the Egyptian way to find: a) $184 \div 8$ b) $21 \div 8$ c) $47 \div 9$

4. a) Show: If n = 3k, then $\frac{2}{n} = \frac{1}{(2k)} + \frac{1}{(6k)}$.

b) Use this decomposition to write $\frac{2}{21}$ as a sum of unit fractions.

5. a) Show: $\frac{2}{n} = \frac{1}{n} + \frac{1}{2n} + \frac{1}{3n} + \frac{1}{6n}$.

b) Use this decomposition to write $\frac{2}{101}$ as a sum of unit fractions.

6. a) Show that if n is a multiple of 5, then $\frac{2}{n}$ can be decomposed into two unit fractions, one of which is $\frac{1}{3n}$.

b) Use this to find a decomposition of $\frac{2}{25}$ and $\frac{2}{40}$ into unit fractions.

7. Use the multi-step method outlined in class to write $\frac{2}{83}$ as a sum of unit fractions..

8. Show that the multi-step method outlined in class for decomposing $\frac{2}{n}$ into unit fractions actually works. (In the notation used in class, suppose $z > \frac{n}{2}$ and that $n - z = a + b + c + d$. Then consider $\frac{1-(\frac{a}{z}+\frac{b}{z}+\frac{c}{z}+\frac{d}{z})}{n} + \frac{1}{z}$.

9. Another method that has been proposed as a way that the Egyptian scribes arrived at their $\frac{2}{n}$ decompositions involves the following steps:

1. Choose a number z > $\frac{n}{2}$ and that has several factors.
2. Find n – z
3. Find z – (n – z)
4. Partition this result into not more than three components that are factors of z.
5. Divide these components by z.
6. Divide these fractions by n.
7. Precede them by $\frac{1}{z}$.

Use this procedure to decompose $\frac{2}{67}$ (let z = 40) and $\frac{2}{91}$ (let z = 70) into a sum of unit fractions.

10. Use the method of false position to solve problem #25 from the Rhind papyrus:

$$x + \left(\frac{1}{2}\right) x = 16$$

11. Use the method of false position to solve:

a) heap + $\frac{1}{4}$ heap = 21

b) $\frac{2}{3}$ heap + 4 heap = 42

12. Solve (any way you can) problem #43 from the Rhind papyrus: Divide 100 loaves among 5 men so that the shares are in arithmetic progression and so that $\frac{1}{7}$ of the sum of the three largest shares is equal to the sum of the two smallest. (An arithmetic progression has the form: a, a + d, a + 2d, a + 3d, ...).

13. Solve problem #41 from the Rhind papyrus: A cylindrical granary has a diameter of 9 cubits and height 10 cubits. What is the amount of grain in it? Use the Egyptian value for π.

14. The Sixth –century Hindu mathematician Aryabhata had the following procedure for finding the area of a circle: half the circumference times half the diameter. How accurate is this rule?

15 a) Show that $\frac{z}{pq} = \frac{1}{pr} + \frac{1}{qr}$ where r = $\frac{(p+q)}{z}$. This method for decomposing a fraction into two unit fractions is indicated on a papyrus dating back to sometime between 500 and 800 CE.

b) Take z = 2, p = 1, and q = 7 to find the decomposition for $\frac{2}{7}$ found in the Rhind papyrus.